Making the equations homogeneous by writing in them $z = 1$,

$$[(x^2+y^2)-(a^2+b^2)z^2]^2\tan^2\varphi = 4z^2(b^2x^2+a^2y-a^2b^2z^2). \qquad (C)$$

$$\frac{\varkappa x}{a^2}+\frac{\lambda y}{b^2} = z. \qquad (D)$$

Eliminating z, the coordinates of the points in which (C) and (D) intersect are given by the equation

$$\left[(x^2+y^2)-(a^2+b^2)\left(\frac{\varkappa x}{a^2}+\frac{\lambda y}{b^2}\right)^2\right]^2\tan^2\varphi = 4\left(\frac{\varkappa x}{a^2}+\frac{\lambda y}{b^2}\right)^2$$
$$\times\left[b^2x^2+a^2y-a^2b^2\left(\frac{\varkappa x}{a^2}+\frac{\lambda y}{b^2}\right)^2\right].$$

Expanding and reducing, we get

$$Ax^4+Bx^3y+Cx^2y^2+Dxy^3+Ey^4 = 0. \qquad (E)$$

Where

$A = b^4[a^2-b^2(a^2+b^2)\varkappa^2]^2\tan^2\varphi-4a^2b^8\varkappa^2+4a^2b^{10}\varkappa^4$,

$B = 4[a^2b^6(a^2+b^2)^2\varkappa^3\lambda-a^4b^4(a^2+b^2)\varkappa\lambda]\tan^2\varphi-8a^4b^6\varkappa\lambda+16a^4b^8\varkappa^3\lambda$,

$C = [2a^4b^4-2a^2b^2(a^2+b^2)(a^4\lambda^2+b^4\varkappa^2)+6a^4b^4(a^2+b^2)^2\varkappa^2\lambda^2]\tan^2\varphi$
$\qquad -4(a^6b^4\lambda^2+a^4b^6\varkappa^2)+24a^6b^6\varkappa^2\lambda^2$,

$D = 4[a^6b^2(a^2+b^2)^2\varkappa\lambda^3-a^4b^4(a^2+b^2)\varkappa\lambda]\tan^2\varphi-8a^6b^4+16a^8b^4\varkappa\lambda^3$,

$E = a^4[b^2-a^2(a^2+b^2)\lambda^2]^2\tan^2\varphi-4a^8b^2\lambda^2+4a^{10}b^2\lambda^4$.

But if (D) is tang't to (C), equation (E) must have equal roots and therefore its discriminant vanishes. Therefore

$$4(12AE-3BD+C^2)^3 = (72ACE+9BCD-27AD^2-27EB^2-2C^3)^2.$$

(Salmons Higher Algebra, 3rd edition, p. 306.)

This is an equation between the coordinates $\varkappa$, λ and is, therefore the required envelope.

NOTE ON AN INDETERMINATE EQN., BY WM. HOOVER, A. M.—The following quotation is from a communication by M. E. Catalan to *Journal de Mathematiques* for August, 1882.

"The identity

$$(a+b)^2(a-2b)^2(b-2a)^2+27a^2b^2 = 4(a^2-ab+b^2)^3,$$

easy of verification, gives an indefinite number of solutions, in entire numbers, of

$$x^2+3y^2 = z^3.$$

"We can take

$x = \frac{1}{2}(a+b)(a-2b)(b-2a$, $y = \frac{3}{2}ab(a-b)$, and $z = a^2-ab+b^2$."